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Novel Scaling Behavior of Directed Polymers: Disorder Distribution with Long Tails

Umberto Marini Bettolo Marconi[1,2] and Yi-Cheng Zhang[1]

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We study the problem of directed polymers (DP) on a square lattice. The distribution of disorder ε is assumed to be independent but non-Gaussian. We show that for distributions with a power-law tail $P(\varepsilon) \sim 1/|\varepsilon|^{1+\mu}$, where $\mu > 2$, so that the mean and variance are well defined, the scaling exponent ν of the DP model depends on μ in a continuous fashion.

KEY WORDS: Directed polymers; non-Gaussian distribution tails.

The directed polymer problem[(1)] (DP) is currently a subject of active research. It is related to various other interesting problems, such as the disorder-driven Burgers equation,[(2)] domain structures of random spin systems,[(3)] and surface growth.[(4)] From the theoretical point of view the DP model is one of the simplest models with quenched disorder. It is analytically tractable, at least on Cayley trees[(5)] and other special lattices.[(6)] One hopes that everything should be obtainable in $1+1$ dimensions. The model is yet rich enough to share many characteristics with the more complex random systems, such as spin glasses. Often the DP model is taken as a testing ground for new ideas. For instance, recently[(7,8)] it has been pointed out that in spite of the fact that the replica symmetry is weakly broken, the replica-symmetric scaling behavior is not affected. The stability of the DP ground states has also been studied[(9)] and it is found that under infinitesimal change of the random environment the DP ground states can have appreciable variations.

In the present work we are concerned with another (in)stability problem relevant to the DP model. Let us consider a disordered system in

[1] INFN and Dipartimento di Fisica, Università di Roma, 00185 Rome, Italy.
[2] Also at Dipartimento di Matematica e Fisica, Università di Camerino, Italy.

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which the energy distribution on each lattice bond (or site) has a well-defined mean and variance, but with some of the higher moments diverging. How does this non-Gaussian distribution influence the scaling behavior?

The model can be defined on a square lattice as follows:

$$Z(x, t) = \sum_{W} \exp -\beta E_W \tag{1}$$

where x and t can be called the space and time variables, W denotes an oriented walk in the t direction, the sum is over all such walks which connect the origin $(0, 0)$ to (x, t), E_W is the sum of the energy ε on the lattice bonds of the walk W, and β is the inverse of temperature.

We perform numerical simulations using the transfer matrix technique, and limit ourselves to the case of zero temperature ($\beta \to \infty$). This amounts to finding the most dominating walk which can be called the ground state in the above path sum (1). For a given random sample we will find a best path of length t starting from the origin which typically deviates in the transverse direction, with respect to the geometrical center $x = 0$, a distance x_c. After averaging over random samples it is expected to be a scaling function of t with an exponent ν, $x_c \sim t^{\nu}$. It is generally believed that zero-temperature scaling behavior extends also to a finite low-temperature region. For the $(1+1)$-dimensional DP problem with *Gaussian* independent disorder, the scaling exponent is known to be $\nu = 2/3$ exactly over the entire temperature region and there is no phase transition in the sense that the specific heat is a smooth function[10] of temperature.

We assume that disorder is independently on each lattice bond, according to the following power-law distribution density:

$$P(\varepsilon) \sim \frac{1}{|\varepsilon|^{1+\mu}}, \quad |\varepsilon| \geqslant 1; \qquad P(\varepsilon) = 0, \quad |\varepsilon| < 1 \tag{2}$$

We consider $\mu > 2$, so that there is a well-defined mean $\langle \varepsilon \rangle$ (zero in the symmetrical case) and variance $\langle \varepsilon^2 \rangle = O(1)$. For $\mu < 2$, (2) is equivalent to the Levy distribution where the second moment is infinite; that problem is interesting in itself, but we will not discuss it here.

The distribution (2) has a so-called long tail; traditionally, not much attention has been paid to these "weak" tails in disorder. There is the general feeling that as long as the variance of microscopic disorder is finite, the scaling behavior of physical observables on macroscopic scales would not be affected by the higher moments of disorder. Take the random walk problem as an example: suppose that a free walker at each step makes a jump of a distance Δx distributed according to $P(\Delta x)$ of (2); the end-to-end distance x after t (t large) steps will scale as $x^2 \sim t$. Normally the most relevant physical quantities are expressed through the second moments of the macroscopic observables, which depend only on the first two moments of microscopic disorder. This fact can be actually summed up in a theorem which extends the above assertion for the random walk model to all Markovian processes.[11] We have not said anything about the higher moments of the macroscopic observables which are surely related to higher moments of disorder, but who would care?

Since the DP problem does not describe a Markovian process, things can be completely different from the above random walk example. The above tails contain singularity large values, so that the optimal paths or ground state will try to include them on their way. This fact may cause larger fluctuations than the ones observed in the presence of Gaussian or truncated disorder.

To check the above hypothesis, we implement the zero-temperature transfer matrix simulation on a 1000 by 1000 square lattice. The above distribution (2) can be realized on a computer, and the energy ε on lattice bonds takes value from

$$\pm \mathrm{ranf}^{-1/\mu} \tag{3}$$

where "ranf" is a system supplied random number generator which provides the uniform distribution between zero and one. Since we are interested in the tail, particular attention has to be paid to the quality of random number generators.

We plot the transverse fluctuation x_c in Fig. 1 for three different values

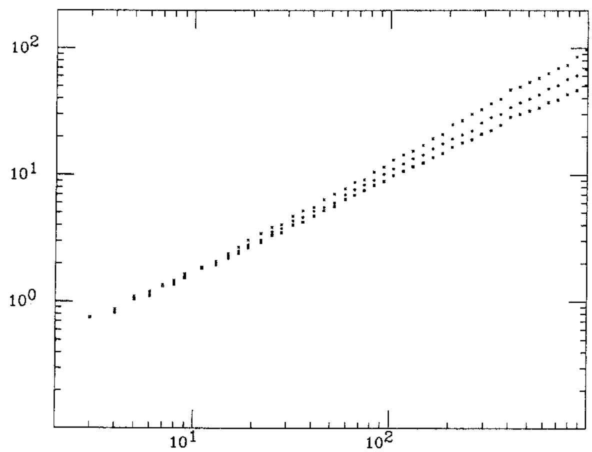

Fig. 1. The transverse fluctuation x_c (vertical axis) against time t for three different values of μ: 2.2, 3, and 4. There are three sets of corresponding data; from top to bottom, they show distinct scaling behaviors.

of μ, where the data were typically averaged over 1000 samples. The data scale quite well and we are able to estimate the scaling exponent $x_c \sim t^\nu$: $\nu = 0.90 \pm 0.03$ for $\mu = 2.2$, $\nu = 0.81 \pm 0.01$ for $\mu = 3$, and $\nu = 0.72 \pm 0.01$ for $\mu = 4$. In Fig. 2 we plot the exponent ν against μ for more values of μ, where we also report two cases $\mu = 1.9$, 2.0 when the second moment is divergent. When $\mu \to 2$ we have noticed larger fluctuations in the data and it takes a longer time to reach the asymptotic regions. This is rather similar to the situation when a traditional critical point is approached. We believe that when $\mu \to 2^+$, $\nu \to 1$; and for $\mu < 2$ (Levy distribution) the polymer exponent ν becomes saturated at 1. However, we cannot be very sure of this, since our algorithm involves only the nearest neighbors; the possibility of having $\nu > 1$ is excluded. As a consequence, the interesting case of the Levy distribution cannot be studied in the present fashion. There appears to be an upper critical value of $\mu_c \approx 6$ above which the scaling exponent returns to its much celebrated Gaussian value $\nu = 2/3$. We have checked this for $\mu = 7$, 10, and 30. We attribute this finding to the fact that there the tails are too weak to spoil the traditional Gaussian scaling behaviors. The existence of the critical μ_c implies a qualitative distinction between the low-μ and high-μ regions.

There is another exponent ω, which is the energy fluctuation.[1] In our simulations we have not yet obtained satisfactory estimates of this exponent. However, we expect the exponent identity

$$\nu = (1 + \omega)/2 \tag{4}$$

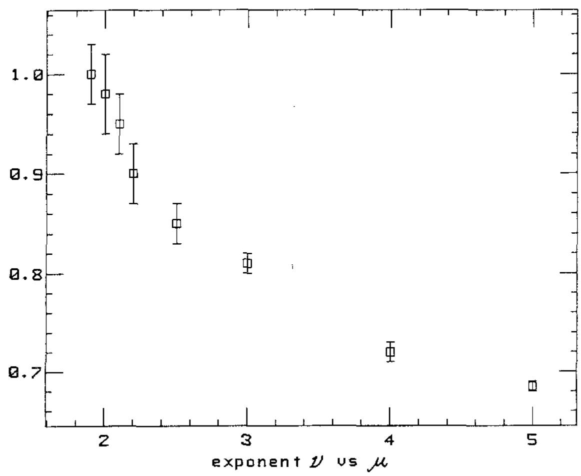

Fig. 2. The scaling exponent ν against μ, for various μ. The errors are from our subjective assesment.

to hold also for disorder with tails, since it can be shown[12] that Eq. (4) is rigorously true for any disorder without temporal correlations. This is due to the Galilean invariance. There is another exact relation which is valid only for Gaussian independent disorder: the fluctuation-dissipation theorem,[3] which asserts that the exponent ν is 2/3 on the basis of invariant distribution measures. It is clear that this theorem is violated in the presence of independent disorder with tails. It remains to be shown if alternatives to the above theorem exist so as to give analytical predictions of the exponent ν in terms of μ.

We believe that the same type of question might be asked for other random systems, for instance, spin glasses and Ising models with random fields or bonds. The disorder tail studied in this work may also change the universality classes there. We wonder if and how the replica symmetry for the $(1+1)$-dimensional DP problem[7,8] maybe broken when disorder has long tails. These tails may also influence some dynamical processes, such as invasion percolation, which shares many features with growth models. Further work along these lines is needed.

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